Northwestern University

Name: ____________________

Math 230, Final

Summer 2012

August 8, 2012

Prob.	Points	Score
1	10	
2	15	
3	5	
4	5	
5	5	
6	10	
7	10	
8	5	
9	10	
10	10	
11	10	
12	10	
13	15	
Total	120	

Instrctions:

- You have 120 minutes to complete the exam.
- You may not use books, notes or calculators.
- Read each question carefully.
- Write legibly and show your work. Feel free to use both sides of each page.
- Ask for additional paper if you need.
- This exam has 13 pages and 13 problems. Please make sure that all pages are included.

GOOD LUCK !

1. (10 points.) Determine whether the statements below are true or false and circle your answer.

(a) If $\overrightarrow{r}(t)$ is a differentiable vector function then $\frac{d}{dt}|\overrightarrow{r}(t)| = |\overrightarrow{r}'(t)|$.

True **False**

(b) For a function $f(x, y)$ if $\lim_{x\to 0} f(x, 0) = \lim_{y\to 0} f(0, y) = 0$ then $\lim_{(x,y)\to(0,0)} f(x, y) = 0$.

True **False**

(c) If for $f(x, y)$, the second order partial derivatives all exist and are continuous then $f_{xy}(x, y) = f_{yx}(x, y)$.

True **False**

(d) If $\overrightarrow{w} \parallel \overrightarrow{v}$, then $\overrightarrow{v} \times (\overrightarrow{u} + \overrightarrow{w}) = \overrightarrow{v} \times \overrightarrow{u}$.

True **False**

(e) If $f(x, y)$ has continuous second order partial derivatives at (a, b), also $\nabla f(a, b) = \langle 0, 0\rangle$ and $f_{xx}(a, b)f_{yy}(a, b) < [f_{xy}(a, b)]^2$, then (a, b) is a saddle point of f.

True **False**

2. (15 points) Consider the following function.

$$f(x,y) = \begin{cases} \frac{x^2y^2}{(x^2+y^2)[(x-1)^2+y^2]} & (x,y) \neq (0,0) \text{ or } (1,0) \\ 0 & (x,y) = (0,0) \\ 1 & (x,y) = (1,0) \end{cases}$$

(a) Find the limit $\lim\limits_{(x,y)\to(0,0)} f(x,y)$.

(b) Evaluate $\lim\limits_{(x,y)\to(1,0)} f(x,y)$.

(c) Find all the points where f is continuous.

3. (5 points) Find an equation for the tangent plane to the surface given by the equation

$$e^{xy+z} \cos(\ln(z)) = 1$$

at the point $(1, -1, 1)$.

4. (5 points) Find and sketch the domain of the following function

$$f(x,y) = \frac{\ln(x^2+y^2-1)}{\sqrt{x+y}}$$

5. (5 points) Write down equations for two level curves of the following function and sketch them, (clearly label the intercepts)

$$f(x,y) = e^{2x^2-y^2}$$

6. (10 points.) Consider the following lines

$$L_1 \; : \; \overrightarrow{r}(t) = \langle 1, -1, 1 \rangle + t\langle 2, 1, -3 \rangle$$

$$L_2 \; : \; \overrightarrow{q}(s) = \langle 0, 0, 0 \rangle + s\langle 1, -2, 1 \rangle$$

(a) Determine whether they are intersecting, parallel or skew.

(b) If they are skew or parallel find the distance between them, if they are intersecting find the point of intersection.

7. (10 points.) A cannon is fired from a hill top which is 200 meters above sea level, at an angle of 30° with the horizontal and an initial speed of 500 m/s.

   (a) Find the acceleration, velocity, and position as vector functions of time using your choice of coordinates. (There is no air drag and acceleration due to gravity g = 10m/s^2)

   (b) At what time does the cannon ball hit the ground which is at sea level?

8. (5 points) Find the tangent lines to the curve

$$x^2 + y^2 + 2xy - x + y = 0$$

at the points $(1, 0)$ and $(0, -1)$.

9. (10 points) Consider the function

$$F(x,y,z) = \begin{cases} \frac{x^3}{x^2+y^2+z^2} & (x,y,z) \neq (0,0,0) \\ 0 & (x,y) = (0,0,0) \end{cases}$$

(a) Find the directional derivative $D_{\overrightarrow{u}}F(-1,1,0)$, where $\overrightarrow{u} = \langle \frac{3}{7}, -\frac{2}{7}, \frac{6}{7} \rangle$ is a unit vector.

(b) Using the limit definition calculate $\dfrac{\partial F}{\partial x}(0,0,0)$.

10. (10 points) Calculate the linear approximation of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at $(3, 2, 6)$, and use it to estimate $\sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2}$.

11. (10 points) Consider the following functions

$$f(x, y) = xe^{xy} \quad x(s, t) \quad y(s, t)$$

Assume $x(0, 1) = 2$, $y(0, 1) = 1$, $\dfrac{\partial x}{\partial t}(0, 1) = 3$ and $\frac{\partial y}{\partial t}(0, 1) = -1$. Compute $\dfrac{\partial g}{\partial t}(0, 1)$ where

$$g(s, t) = f(x(s, t), y(s, t))$$

12. (10 points) Find and determine the nature of critical points of the function

$$x^2 + y^3 - 6xy + 3x + 6y$$

13. (15 points) Find the maximum and minimum of the function $f(x, y) = x^3 + y^2 - 6x^2$, in the closed and bounded elliptic region $D = \{(x, y) \mid 3x^2 + y^2 \leq 3\}$.